Optimal Bidding Strategy for a Generation Company Under Price Uncertainty

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ABSTRACT
Considering market clearing price uncertainty and based on mixed-integer programming and stochastic programming, a bidding model of bidding output independent on scenario for a thermal power producer is presented. This model can give nondecreasing bidding curves to meet the market requirements. Simulations for a generation company’s bidding behavior are performed using the model. Results show that the proposed model can provide adequate information to bidding curves.

Keywords: electricity markets; bidding strategy; mixed-integer programming; stochastic programming; price uncertainty.

1. INTRODUCTION
In the deregulated electricity markets, a generation company (GENCO) bids power production to the market operator to maximize its profit. Thus, it is very crucial for a GENCO to devise a good bidding strategy[1]. There are basically two classes of approaches to developing bidding strategies, equilibrium models and non-equilibrium models. Equilibrium models such as supply function equilibrium and Cournot equilibrium were widely applied for developing GENCOs’ bidding strategies and analyzing market power in electricity markets[2, 3]. However, unit constraints such as minimum on/off time, ramping limits, and startup/shutdown cost were not considered in most of the equilibrium models. Non-equilibrium approaches in the literature for developing optimal bidding strategies have been widely researched. [4] proposed an ordinal optimization method which used an approximate model for analyzing the impact of a GENCO’s bidding strategies on market clearing price. Deterministic price-based unit commitment (PBUC) was presented for developing bidding strategies in [5-7]. However, the precision of market price forecasting would have a significant impact on solution. Due to uncertainty of electricity markets, it is difficult to forecast market prices accurately. In [8], the market price uncertainty was taken into account while developing optimal bidding strategies in multi-markets, which consists of day-ahead market, reserve market.

Considering market clearing price uncertainty and based on mixed-integer programming and stochastic programming, a bidding model of bidding output independent on scenario for GENCO is presented. The problem is formulated as a stochastic mixed integer linear programming and solved by commercial MIP solver. The rest of this paper is organized as follows: a bidding model is provided in Section 2. Section 3 describes the scenario approach. Section 4 gives results from case studies. Finally, conclusions are drawn in Section 5.

2. BIDDING MODEL
Consider a pool-based electricity market in which GENCOs submit day-ahead bids for each hour before closure of the market. GENCOs are assumed to be price takers [2], there is no interference among bids offered by all units in a GENCO. Therefore this paper only focuses a GENCO with one unit.

2.1 Objective Function for a GENCO
The objective function is to maximize a GENCO’s expected profits as follows

\[
\max \sum_{m} \sum_{t} \pi_m \lambda_m \Delta t \Delta t - C(p_m) \tag{1}
\]

where \( t \) is bidding period, running from 1 to \( T \); \( \lambda_m \) is the price in scenario \( m \) in period \( t \); \( \pi_m \) is the probability of scenario \( m \); \( p_m \) is the bidding power output in scenario \( m \) in period \( t \); \( \lambda_m \) is the unit state in period \( t \), which is a binary variable. \( C(p_m) \) is the generation cost in scenario \( m \) in period \( t \), it consists of three parts: fuel cost \( (C_f m) \), startup cost \( (C_s) \) and shutdown cost \( (C_s) \).

2.1.1 Fuel Cost
In general, fuel cost is expressed by a quadratic function as follows:

\[
C_f m = a(p_m)^2 + bp_m + c \tag{2}
\]

where \( a \), \( b \) and \( c \) are the coefficients of the quadratic production cost function; The fuel cost is modeled using a piecewise linear function [9]. If enough piecewise blocks are considered, this approximation is accurate. A mixed-integer linear formulation of this approximation is given by (3)-(8):

\[
C_f m = C_f u_m + \sum_{i=1}^{S} \delta_{i,m} \Delta \lambda_{i,m}, \forall m \tag{3}
\]

\[
p_m = \sum_{i=1}^{S} \delta_{i,m} \lambda_{i,m} + \mu_m u_m, \forall m \tag{4}
\]

\[
\delta_{i,m} \mu_m \leq \delta_{i,m}, \forall m \tag{5}
\]
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\[ \delta_{i}^{m} \leq \delta_{i} - \delta_{i+1}, \quad \forall l=2, \ldots, L-1 \]  
\[ \delta_{i}^{m} \leq p_{\text{max}} - \delta_{i+1}, \quad \forall m \]  
\[ \delta_{i}^{m} \geq 0, \quad \forall m \]  

where \( C^{f} \) is fixed cost, and \( C^{f} = a(p_{\text{max}})^{2} + bp_{\text{max}} + c \). L is the number of blocks of the piecewise linear production cost function; \( \delta_{i}^{m} \) is the power output through block \( l \) of the piecewise linear production cost function in scenario \( m \) in period \( r \); \( s_{r} \) is the slope of block \( l \) of the piecewise linear production cost function; \( \delta_{i} \) is the piecewise point.

2.1.2 Startup Cost

When the unit state changes from off to on, startup cost occurs and is denoted by fixed cost as follows,

\[ C_{i}^{u} = \begin{cases} C^{f} & \text{if } u = 1, \ u_{i-1} = 0 \\ 0 & \text{else} \end{cases} \]  

Note that the expressions in (9) is nonlinear, linearization of (9) is expressed by \[9\]

\[ C_{i}^{u} \geq C^{f} (u_{i} - u_{i-1}) \]  
\[ C_{i}^{u} \geq 0 \]

2.1.3 Shutdown Cost

When the unit state changes from on to off, shutdown cost occurs and is denoted by fixed cost as follows,

\[ C_{i}^{d} = \begin{cases} C^{f} & \text{if } u = 0, \ u_{i-1} = 1 \\ 0 & \text{else} \end{cases} \]  

the linearization of (12) is expressed as follows:

\[ C_{i}^{d} \geq C^{f} (u_{i-1} - u_{i}) \]  
\[ C_{i}^{d} \geq 0 \]

2.2 Constraints

This bidding problem is subjected to a variety of unit constraints, which include generation constraints, ramping rate constraints, minimum up and down time constraints and nondecreasing bidding curves constraints.

2.2.1 Generation constraints

In each period and each scenario, generation of unit must be within its generation limits

\[ p_{\text{min}} u_{i} \leq p_{i}^{m} \leq \bar{p}_{i}^{m}, \quad \forall m \]  
\[ 0 \leq p_{i}^{m} \leq p_{\text{max}} u_{i}, \quad \forall m \]  

where \( p_{\text{max}} \) and \( p_{\text{min}} \) are the upper and lower limits of unit; \( \bar{p}_{i}^{m} \) is the maximum output of unit in scenario \( m \) and period \( t \). If \( u_{i} = 1 \), the generation unit is online and \( u_{i} = 0 \) otherwise; In addition, generation of unit is subjected to the Startup ramp limit \((\Delta^{SU})\) and Shutdown ramp limit \((\Delta^{SD})\) [9], which are given by below

\[ \bar{p}_{i}^{m} \leq \begin{cases} \Delta^{SU} & \text{if } u_{i} = 1, u_{i-1} = 0 \\ \Delta^{SD} & \text{if } u_{i} = 1, u_{i-1} = 0, \forall m \end{cases} \]  
\[ \min (p_{i}^{m} + \Delta_{s} + \bar{p}_{\text{max}}) \]  

where \( \Delta_{s} \) is ramping up limit. The linear formulation of (17) after linearization is shown in (18) and (19)

\[ \bar{p}_{i}^{m} \leq p_{i}^{m} + \Delta_{s}, \quad \forall m \]  
\[ p_{i}^{m} \leq p_{i}^{m} + \Delta_{s}, \quad \forall m \]  

2.2.2 Ramping rate constraints

In each period and each scenario, generation of unit must not exceed its maximum ramping limits

\[ p_{i+1}^{m} - p_{i}^{m} \leq \Delta_{r}, \quad \forall m \]  
\[ p_{i}^{m} - p_{i-1}^{m} \leq \Delta_{r}, \quad \forall m \]  

where \( \Delta_{r} \) is ramping down limit. The linear formulation of (20) and (21) after linearization is shown in (22) and (23), respectively.

\[ p_{i+1}^{m} - p_{i}^{m} \leq \Delta_{s} \Delta_{r}, \quad \forall m \]  
\[ p_{i}^{m} - p_{i-1}^{m} \leq \Delta_{s} \Delta_{r}, \quad \forall m \]

2.2.3 Minimum-up time constraints

As shown in [5] and [10], a computationally efficient way to model the minimum-up time constraint is as follows:

\[ \sum_{k=1}^{T} u_{ik} \geq \tau_{\text{min}}^{u} \]  
\[ \sum_{k=1}^{T} [u_{ik} - (u_{i-1} - u_{i})] \geq 0, \quad \forall t = T - \tau_{\text{min}}^{u} + 2, \ldots, T \]  
\[ \sum_{k=1}^{T} (1 - u_{ik}) = 0 \]

The first condition enforces the minimum-up time constraint from \( t=\tau_{\text{min}}^{u} + 1 \) to \( T - \tau_{\text{min}}^{u} + 1 \). The second condition enforces the minimum-up time constraint for the rest of the planning horizon such that if the unit started at any of these periods, it remains on until the end of the planning horizon. The last condition ensures that the thermal unit will not be shut off unless it has been on initially for a sufficient number of periods (\( \tau_{\text{min}}^{u} \)).

2.2.4 Minimum-down time constraints

Similarly, the Minimum-down time constraints can be enforced as follows:

\[ \sum_{k=1}^{T} (1 - u_{ik}) \geq \tau_{\text{min}}^{d} \]  
\[ \sum_{k=1}^{T} [1 - u_{ik} - (u_{i-1} - u_{i})] \geq 0, \quad \forall t = T - \tau_{\text{min}}^{d} + 2, \ldots, T \]
2.2.5 Nondecreasing bidding curves constraints

Most energy markets require hourly bidding offers that are nondecreasing with respect to energy market prices [11]. This requirement can be enforced in the optimization constraints as follows:

\[(\lambda_{i}^{m} - \lambda_{i}^{m'}) (p_{i}^{m} - p_{i}^{m'}) \geq 0, \forall m, m'\]  \hspace{1cm} (30)

\[p_{i}^{m} = p_{i}^{m'}, \forall m, m': \lambda_{i}^{m} = \lambda_{i}^{m'}\]  \hspace{1cm} (31)

(30) enforces nondecreasing offers, and (31) enforces the fact that scenarios with the same price must have the same bidding offer.

3. PRICE SCENARIOS

Due to the uncertainty of markets, market price behaviors as a random variable. We assume that it follows normal distribution[1], and can be obtained by forecasting tools. Then, Monte Carlo simulation is executed \(M\) (a large number) times to generate scenarios for market prices when the probability of each scenario is assumed to be \(1/M\). Figure 1 shows the diagram of the scenario tree.

![Diagram of scenarios tree](image)

If we execute Monte Carlo simulation 50000 times, we could obtain 50000 scenarios, and the resulting stochastic program would be too large to solve [2]. Accordingly, scenario reduction techniques are employed to reduce the number of scenarios in consideration while maintaining a good approximation of the statistical properties of market prices. This paper adopts a reduction method called simultaneous backward reduction as proposed in [12].

4. CASE STUDY

In this section, simulations are performed on a unit from [5]. The data for the unit is given in Table 1. A coal-fired unit has been chosen. Its original characteristics have been altered so that the following examples show the differences between a typical formulation and a precise formulation as the one proposed in this paper. Thus, ramp up and down rate limits have been decreased and start-up and shut-down ramp rates have been added. The fuel cost is modeled through 10 blocks as shown in Table 2. The means of price and the 30 scenarios (10000 scenarios before reduction) are shown in Figure 1.

![Figure 1. Diagram of scenarios tree](image)

![Figure 2. Means of price and 30 scenarios](image)

Table 1. Unit characteristics

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p_{\text{max}} / \text{MW})</td>
<td>294</td>
<td>(r_{\text{min}} / \text{h})</td>
<td>4</td>
</tr>
<tr>
<td>(p_{\text{min}} / \text{MW})</td>
<td>112</td>
<td>(p_{0} / \text{MW})</td>
<td>112</td>
</tr>
<tr>
<td>(\Delta_{\text{up}} / \text{MW})</td>
<td>170</td>
<td>(r_{\text{up}} / \text{h})</td>
<td>1</td>
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<tr>
<td>(\Delta_{\text{down}} / \text{MW})</td>
<td>160</td>
<td>(C^f / $)</td>
<td>120</td>
</tr>
<tr>
<td>(\Delta_{\text{down}} / \text{MW/h})</td>
<td>60</td>
<td>(C^f / $)</td>
<td>4480</td>
</tr>
<tr>
<td>(\Delta_{\text{up}} / \text{MW/h})</td>
<td>50</td>
<td>(C^f / $)</td>
<td>130</td>
</tr>
<tr>
<td>(\varepsilon_{\text{min}} / \text{h})</td>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Piecewise linear variable cost

<table>
<thead>
<tr>
<th>Index</th>
<th>slope$/\text{MWh}$</th>
<th>piecewise point/MW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>48.0</td>
<td>130</td>
</tr>
<tr>
<td>2</td>
<td>49.0</td>
<td>148</td>
</tr>
<tr>
<td>3</td>
<td>50.5</td>
<td>166</td>
</tr>
<tr>
<td>4</td>
<td>51.5</td>
<td>184</td>
</tr>
<tr>
<td>5</td>
<td>53.0</td>
<td>202</td>
</tr>
<tr>
<td>6</td>
<td>54.5</td>
<td>220</td>
</tr>
<tr>
<td>7</td>
<td>55.5</td>
<td>238</td>
</tr>
<tr>
<td>8</td>
<td>57.0</td>
<td>256</td>
</tr>
<tr>
<td>9</td>
<td>58.5</td>
<td>274</td>
</tr>
<tr>
<td>10</td>
<td>59.0</td>
<td>294</td>
</tr>
</tbody>
</table>

Table 3 presents the profits in 30 scenarios. The scenario with the least profit is number 19. Unit state is 111111111111111111100000.
Table 3 profit with different scenarios

<table>
<thead>
<tr>
<th>scenario</th>
<th>profit/$</th>
<th>scenario</th>
<th>profit/$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5771.32</td>
<td>16</td>
<td>7110.12</td>
</tr>
<tr>
<td>2</td>
<td>7209.27</td>
<td>17</td>
<td>2264.73</td>
</tr>
<tr>
<td>3</td>
<td>8825.63</td>
<td>18</td>
<td>6487.09</td>
</tr>
<tr>
<td>4</td>
<td>2964.80</td>
<td>19</td>
<td>1284.16</td>
</tr>
<tr>
<td>5</td>
<td>6349.97</td>
<td>20</td>
<td>5694.78</td>
</tr>
<tr>
<td>6</td>
<td>5827.62</td>
<td>21</td>
<td>5518.82</td>
</tr>
<tr>
<td>7</td>
<td>8124.76</td>
<td>22</td>
<td>2569.24</td>
</tr>
<tr>
<td>8</td>
<td>4362.03</td>
<td>23</td>
<td>7247.59</td>
</tr>
<tr>
<td>9</td>
<td>4886.03</td>
<td>24</td>
<td>7502.52</td>
</tr>
<tr>
<td>10</td>
<td>6432.09</td>
<td>25</td>
<td>4270.21</td>
</tr>
<tr>
<td>11</td>
<td>4655.04</td>
<td>26</td>
<td>8267.01</td>
</tr>
<tr>
<td>12</td>
<td>4893.15</td>
<td>27</td>
<td>7516.29</td>
</tr>
<tr>
<td>13</td>
<td>7764.48</td>
<td>28</td>
<td>8419.90</td>
</tr>
<tr>
<td>14</td>
<td>5270.22</td>
<td>29</td>
<td>7094.86</td>
</tr>
<tr>
<td>15</td>
<td>4796.94</td>
<td>30</td>
<td>6994.13</td>
</tr>
</tbody>
</table>

Taking period 6, 1 and 9 for example, bidding curves are obtained using the method proposed in this paper, and shown in Figure 3, 4 and 5, respectively. From Figure 3, when the price increases, the generation output is nondecreasing, and outputs are different with respect to different price scenarios.

![Figure 3 Bidding curve in period 6](image)

From Figure 4, the generation output is always equal to the minimum generation limit of unit. This is because the smallest slope in table 2 is 48 $/MWh, which is greater than the prices in 30 scenarios. In Figure 5, bidding output increases significantly more than that in period 1, this is because the price is higher in period 9, more bidding output will lead to more profit.

![Figure 4 Bidding curve in period 1](image)

![Figure 5 Bidding curve in period 9](image)

5. CONCLUSIONS
This paper presents a bidding model of bidding output independent on scenario considering market clearing price uncertainty and based on mixed-integer programming and stochastic programming. This model can give nondecreasing bidding curves to meet the market requirements. It is worth mentioning that the work uses hourly based forecasts and can be employed for hourly based bidding. Future work may consider the study of the bidding problem with volatile renewable resources.

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REFERENCES


