An Improved Particle Swarm Optimization Algorithm for Dynamic Economic Dispatch Problems

Haifeng Zhang
School of Mechanical Engineering, Shanghai University of Engineering Science, Shanghai, China,

ABSTRACT
In this paper, an improved particle swarm optimization (PSO) algorithm is applied to solve the dynamic economic dispatch (DED) problem considering various generator constraints. A feasible region adjustment strategy is presented to ensure the feasibility of the solution. In order to verify the performance of the approach, the proposed approach is tested with a power system case consisting of 6 thermal units. Results show that the improved PSO approach is effective.

Keywords
particle swarm optimization; dynamic economic dispatch; feasible region adjustment strategy; spinning reserve.

1. INTRODUCTION
The purpose of DED problem is to determine the optimal generation scheme to meet the predicted load demand over a time horizon satisfying the constraint such as ramp-rate limits of generators between time intervals [1-3]. This problem belongs to a high-dimension and nonconvex optimization problem, which is very difficult to find analytical solutions. In recent decades, many salient approaches have been developed to solve such problems, such as genetic algorithm [4, 5], differential evolution [6], and PSO algorithms [7-12].

However, it is not always effective to solve such a problem with equality and inequality constraints using basic PSO, the solutions which satisfy the inequality constraints usually violate the equality constraints. To achieve this goal, a feasible region adjustment strategy is presented to ensure that the solutions satisfy both the inequality and equality constraints. In order to verify the performance of the approach, the proposed approach is tested with a power system case consisting of 6 thermal units. Results show that the improved PSO approach is effective.

2. PROBLEM FORMULATION

2.1 Objective Function
The DED model with the valve point effect usually takes the following form [4, 5]:

\[
\min f_{\text{cost}} = \sum_{t=1}^{T} \sum_{i=1}^{I} [C_i(p_{i,t}) + E_i(p_{i,t})]
\]

where

- \( p_{i,t} \) is the output power (MW) of the \( i \)th unit corresponding to time period \( t \).
- \( C_i(p_{i,t}) \) is the generation cost of the \( i \)th unit corresponding to time period \( t \).
- \( E_i(p_{i,t}) \) is the valve point loading effect of the \( i \)th unit corresponding to time period \( t \).

For the thermal units, the generation cost can be approximated to be a quadratic function of the power output, which is practical for most of the cases, and is expressed by

\[
C_i(p_{i,t}) = a_ip_{i,t}^2 + b_ip_{i,t} + c_i
\]

where \( a_i \), \( b_i \) and \( c_i \) are cost coefficients for the \( i \)th unit.

\[
E_i(p_{i,t}) = e_i \sin(f_i(p_{\text{min},i} - p_{i,t}))
\]

where \( e_i \) and \( f_i \) are coefficients related to valve point effect of the \( i \)th unit. \( p_{\text{min},i} \) is the minimum generation limit of unit \( i \).

2.2 System and Unit Constraints
This DED problem is subjected to a variety of system and unit constraints, which include power balance constraints, generation limits of units, ramp rate limits and spinning reserve constraints. These constraints are discussed below.

2.2.1 power balance constraints
Total power generation must equal the load demand \( P_{d,t} \) in all time period

\[
\sum_{i=1}^{I} p_{i,t} = P_{d,t}
\]

2.2.2 Generation limits of thermal units
The output of each thermal unit must lie in between a lower and an upper bound. These constraints are represented as follows:

\[
p_{\text{min},i} \leq p_{i,t} \leq p_{\text{max},i}
\]

where \( p_{\text{max},i} \) is the maximum generation limit of thermal unit \( i \).

2.2.3 Ramp rate limits of thermal units
The ramp rate limits restrict the operating range of all the units for adjusting the generation between two periods. The generation may increase or decrease within the up and down ramp rate limits as shown below:

\[
-\Delta_{i,d} \times T_{60} \leq p_{i,t} - p_{i,t-1} \leq \Delta_{i,a} \times T_{60}
\]
where \( p_{i,t-1} \) is the output of unit \( i \) at time \( t-1 \), and \( \Delta_{u,i} \) and \( \Delta_{d,i} \) are the upper and lower ramp rate limits, respectively. \( T_{10} \) is the operating period, i.e. 1h.

### 2.2.4 Spinning reserve constraints

The spinning reserve is supplied by the ramping capacity of units and supports the forecast errors in load; spinning reserve constraints are formulated as follows:

\[
\sum_{i} \min(p_{i}\text{max} - p_{i}\text{min}, \Delta_{u,i} \times T_{10}) \geq p_{r,i}
\]

where \( p_{r,i} \) is the reserve level to support forecast error in demand. \( T_{10} \) is 10 minutes. \( p_{i}\text{max} \) and \( p_{i}\text{min} \) are upper and lower generation limits of unit \( i \) including ramp rate limits at time \( t \), and \( p_{i}\text{max} = \min(p_{i}\text{max}, p_{i,t-1} + \Delta_{u,i}) \), \( p_{i}\text{min} = \max(p_{i}\text{min}, p_{i,t-1} - \Delta_{d,i}) \).

### 3. IMPROVED PSO ALGORITHM

#### 3.1 Overview of PSO

PSO, first introduced by Kennedy and Eberhart, is a population-based optimization technique, and conducts its search using a population of particles [7, 8]. Each particle is a candidate solution to the problem and is moved toward the optimal point by adding a velocity with its position. The position and the velocity of the \( j \)th particle in the \( D \) dimensional search space can be expressed as

\[
Y_j = [y_{j1}, y_{j2}, \cdots, y_{jd}]^T \quad \text{and} \quad V_j = [v_{j1}, v_{j2}, \cdots, v_{jd}]^T ,
\]

respectively.

Each particle has its own best position (\( \text{pbest}_j, j = 1,2,\ldots,J \)) and the global best position (\( \text{gbest}^k \)), which represents the best particle found so far at generation \( k \). The global best particle is denoted by \( \text{gbest}^k \), which represents the best particle found so far at generation \( k \) in the whole population. The new velocity and position of each particle at generation \( k+1 \) are calculated as shown below:

\[
V_{j,i}^{k+1} = \omega(k) \cdot V_{j,i}^k + \varphi_1(k) \cdot \text{rand}_1 \cdot (\text{pbest}_j - Y_j) + \varphi_2(k) \cdot \text{rand}_2 \cdot (\text{gbest}^k - Y_j)
\]

\[
Y_{j,i}^{k+1} = Y_{j,i}^k + V_{j,i}^{k+1} , 1 \leq j \leq J
\]

where \( J \) is the population size; \( \omega(k) \) is the dynamic inertia weight factor, and can be dynamically set with the following equation [6]:

\[
\omega(k) = \omega_{\text{max}} - (\omega_{\text{max}} - \omega_{\text{min}}) \cdot k / K
\]

\( \omega_{\text{max}} \) and \( \omega_{\text{min}} \) are initial and final inertia weight factors and set to 0.9 and 0.4 respectively, \( K \) is the maximum number of iteration. \( \varphi_1(k) \) and \( \varphi_2(k) \) are time-varying acceleration coefficients corresponding to cognitive and social behavior [6], and are set with the following equations shown in (11) and (12):

\[
\varphi_1(k) = \varphi_{\text{ini}} + (\varphi_{\text{fin}} - \varphi_{\text{ini}}) \cdot k / K
\]

\[
\varphi_2(k) = \varphi_{\text{ini}} + (\varphi_{\text{fin}} - \varphi_{\text{ini}}) \cdot k / K
\]

\( \varphi_{\text{ini}} \) and \( \varphi_{\text{fin}} \) are the initial values of \( \varphi_1(k) \) and \( \varphi_2(k) \) and are set to 2.5 and 0.5 respectively; \( \varphi_{\text{fin}} \), \( \varphi_{\text{ini}} \) are the final values of \( \varphi_1(k) \) and \( \varphi_2(k) \), and are set to 0.5 and 2.5 respectively.

#### 3.2 Feasible Region Adjustment Strategy

Rewrite the position of the \( j \)th particle as the following matrix:

\[
Y_j = \begin{bmatrix}
y_{j1} & y_{j2} & \cdots & y_{jT} \\
y_{j1} & y_{j2} & \cdots & y_{jT} \\
\vdots & \vdots & \ddots & \vdots \\
y_{j1} & y_{j2} & \cdots & y_{jT} \\
\end{bmatrix} = \begin{bmatrix}
p_{i1} & p_{i2} & \cdots & p_{iT} \\
p_{i1} & p_{i2} & \cdots & p_{iT} \\
\vdots & \vdots & \ddots & \vdots \\
p_{i1} & p_{i2} & \cdots & p_{iT} \\
\end{bmatrix}
\]

where \( I \) is the number of thermal units, and \( I \cdot T = D \). The element of matrix \( y_{ij} \) is the output of the \( i \)th generator at time \( t \). The column vectors represent the output of the individual generator each hour within 24h.

As we all know, the sum of the output of individual generator (i.e. power supply) in each hour must be equal to the demand, i.e.

\[
\sum_{i=1}^{I} p_{ij} = d_{ij}, \quad t = 1,2,\ldots,T.
\]

If the power supply in each hour is greater or smaller than the demand, the feasible region adjustment strategy is applied to the \( j \)th particle, and is described in detail as follows:

1. if \( \sum_{i=1}^{I} p_{ij} < d_{ij} \)
2. if \( \sum_{i=1}^{I} p_{ij} > d_{ij} \)

Under this condition, we use the following equation to adjust the output of the thermal units as shown in (13):

\[
p_{ij}^* = p_{ij} + \frac{p_{ij}^{\text{max}} - p_{ij}^{\text{min}}}{\sum_{i} p_{ij}^{\text{max}} - \sum_{i} p_{ij}^{\text{min}}} \left( d_{ij} - \sum_{i} p_{ij} \right)
\]

We use the following equation (14) to adjust the output of the thermal units and wind farm respectively:

\[
p_{ij}^* = p_{ij} - \frac{p_{ij}^{\text{max}} - p_{ij}^{\text{min}}}{\sum_{i} p_{ij}^{\text{max}} - \sum_{i} p_{ij}^{\text{min}}} \left( \sum_{i} p_{ij} - d_{ij} \right)
\]

#### 3.3 Procedure of Improved PSO Approach

The procedures for implementing the PSO approach are given by the following steps:

**Step 1:** Initialize the parameters, such as population size \( J = 40 \), the maximum iteration number. Set the sequence number of iteration \( k = 1 \).

**Step 2:** Create a swarm of particles as the initial population, including random position and velocity. For any particle which violates the equality constraints, the feasible region adjustment strategy is utilized. Evaluate the fitness of particles and obtain the initial \( \text{gbest}^0 \) and \( \text{pbest}^0 \), \( j = 1,2,\ldots,J \).

**Step 3:** Calculate \( \omega(k) \), \( \varphi_1(k) \) and \( \varphi_2(k) \), then update the position and velocity of each particle in the population according to equations (25) and (26). FAR strategy is applied to any particle which violates the equality constraints.

**Step 4:** Evaluate the fitness of particles and update \( \text{pbest}^t_j, j = 1,2,\ldots,J \) and \( \text{gbest}^t \) of the population.
Step 5: \( k = k + 1 \), if \( k > K \), stop the algorithm and output the global best solution \( (gbest)\) with the best fitness value; otherwise, go back to step 3.

4. CASE STUDY
In order to verify the effectiveness of the proposed approach, a test system with 6 thermal units [7] is employed in this paper. The improved PSO approach has been implemented on a personal computer with 4 processors at 2.93 GHz and 4GB of RAM memory using Matlab 7.9.0.

In this test system, we perform 50 trials using the improved approach considering maximum iteration number is 100, 500, 1000 respectively. The average cost and the average CPU time are listed in Table 1.

**Table 1. The average cost and time**

<table>
<thead>
<tr>
<th>Iteration number</th>
<th>Cost/$</th>
<th>Time/second</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>283290.23</td>
<td>0.83</td>
</tr>
<tr>
<td>500</td>
<td>282009.59</td>
<td>3.18</td>
</tr>
<tr>
<td>1000</td>
<td>281322.81</td>
<td>6.13</td>
</tr>
</tbody>
</table>

From Table 1, we can learn the average cost decreases as the iteration number increases, however, larger iteration number leads to much more computation time. In this paper, we choose 500 as the iteration number for the next simulations.

We choose the solutions with minimum generation cost from 50 trials as the optimal solutions. The optimal solutions (Power output at each period for each unit) are shown in Figure 1.

![Figure 1. Power output of units](image)

5. CONCLUSIONS
This paper presents an improved particle swarm optimization algorithm to solve dynamic economic dispatch problems. A feasible region adjustment strategy is proposed to ensure that the solutions are feasible. Simulation results show that the improved approach could be used as a reliable tool for solving dynamic economic dispatch problems with generator constraints.

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REFERENCES