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GENERAL MATHEMATICAL MODEL FOR INVESTIGATION OF CYLINDRICAL AND CONICAL WORMS, WORM GEARS AND FACE GEARS

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ABSTRACT
The objective of this publication is based on the results of kinematical geometry and toothing theory generalization of geometric correct production of worm surfaces (e.g. turning, milling, grinding), production geometry analysis of tools, the mathematical defining of the geometric and connection relation. Our purpose is to be able to define every thread surface ion one common system so that they could be produced in a modern manufacturing system.

Keywords
conical worms, face gear, hob

1. INTRODUCTION
One the most modern types of cylindrical helicoidal surfaces is the worm generated using a circular profile tool.

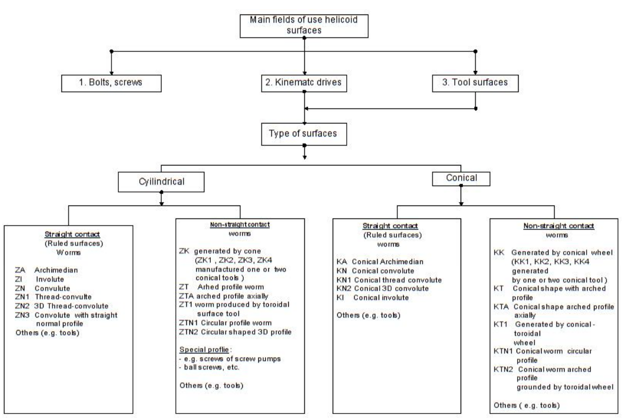


Figure 1. Most frequent applications of helicoidal surfaces [3, 4]

The investigation of geometric problems related to the manufacture of cylindrical helicoid surfaces is best carried out in a general system. This general system makes it possible to discuss cylindrical worm surfaces, their different manufacturing methods and generating tools.

2. INVESTIGATION OF GEOMETRIC PROBLEMS WHEN MANUFACTURING CYLINDRICAL HELICOID SURFACES USING GENERAL MATHEMATICAL-KINEMATIC MODELS
First, $\underline{P}_{1h}$, the general transformation matrix, should be determined, thus solving the connection between the coordinate systems belonging to the generating tool and the generated surface, including the kinematics of generation. Coordinate systems used in this investigation and their relationships are shown in Figure 2.

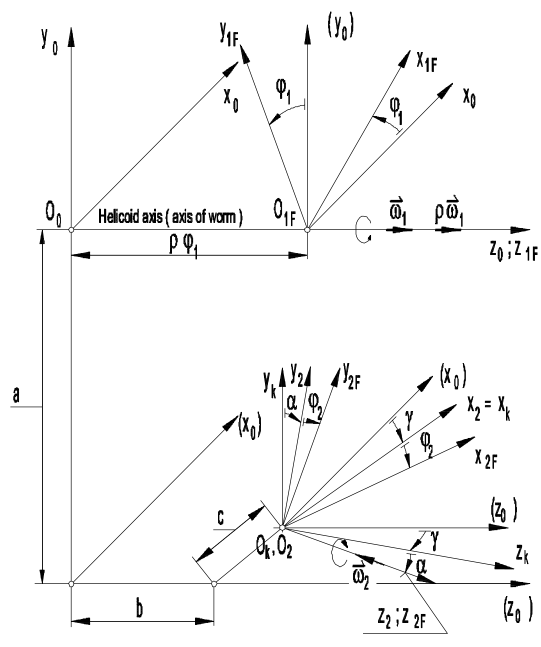


Figure 2: Theory of manufacturing processes during general investigation: relative positions of cylindrical helicoid surfaces

During our investigations the kinematics of generation was handled so that the helicoid surface followed a thread path and the tool surface performed a rotary motion on the left side of the thread profiles; the lead of thread and generator curve together with signs should be taken into consideration.

When discussing geometric problems of manufacturing in general it is necessary to determine generally valid rules for generation of the cylindrical thread surface [5].

The position vector $\vec{r}_g$ of the generating curve in the coordinate system $K_S(\xi,\eta,\zeta)$ is given as a function. This generating curve can be the edge of tool (eg in lathe turning) or the contact curve (eg in grinding) [4]. To formulate the equation of the generating curve, from the practical point of view let the parameter η be chosen as an independent variable. In this way the parametric vector function of the generating curve is found to be:

$$\vec{r}_g = \xi(\eta)\vec{i} + \eta\,\vec{j} + \xi(\eta)\vec{k}\,. \tag{1}$$

The generating curve parametric equation $\vec{r}_g$ is carried by the generating curve parametric equation $K_S(\xi,\eta,\zeta)$ the coordinate system is forced on the thread path along axis ζ with parameter p, so the generating curve will describe a thread surface in coordinate system $K_{1F}(x_{1F}, y_{1F}, z_{1F})$ which, performed before this movement, coincides with the K_S coordinate system (see Figure 3).

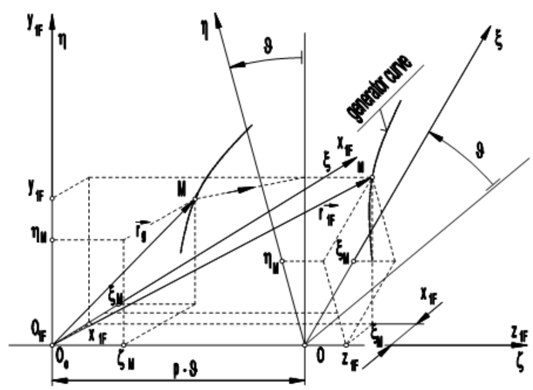


Figure 3: The generating curve of the thread surface in K_{1F} coordinate system

The thread surface described by the generating curve $\vec{r}_g$ can be determined in $K_{1F}(x_{1F}, y_{1F}, z_{1F})$ coordinate system as:

$$\vec{r}_{1F} = \underline{\mathbf{M}}_{1F,S}\cdot\vec{r}_g\,, \tag{2}$$

$$\underline{\mathbf{M}}_{1F,s} = \begin{bmatrix} \cos\vartheta & -\sin\vartheta & 0 & 0 \\ \sin\vartheta & \cos\vartheta & 0 & 0 \\ 0 & 0 & 1 & p\cdot\vartheta \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{3}$$

Therefore, the general equation of the cylindrical worm surface is:

$$\left.\begin{aligned} x_{1F} &= \xi(\eta)\cdot\cos\vartheta - \eta\cdot\sin\vartheta \\ y_{1F} &= \xi(\eta)\cdot\sin\vartheta + \eta\cdot\cos\vartheta \\ z_{1F} &= \xi(\eta) + p\cdot\vartheta \end{aligned}\right\}. \tag{4}$$

It can be seen from the structure of transformation matrix $\underline{M}_{1F,S}$ and the general basically equation of the worm surface (3) that the generating curve $\vec{r}_g$ and worm parameter p determine the worm surface.

The generator curve $\vec{r}_g$ has a decisive role in the case of tool surface generation too. During generation of the tool surface, the generator curve can be the meridian curve or the contact curve. In this case the $\vec{r}_{gsz}$ curve is interpreted in the $K_{20}(x_{20}, y_{20}, z_{20})$ coordinate system using y_{20} as a parameter, so its form is:

$$\vec{r}_{gsz} = x_{20}(y_{20})\,\vec{i} + y_{20}\,\vec{j} + z_{20}(y_{20})\,\vec{k}\,. \tag{5}$$

Rotating $\vec{r}_{gsz}$ generating curve with the $K_{20}(x_{20}, y_{20}, z_{20})$ coordinate system round the z_{20} axis the $\vec{r}_{gsz}$ curve will describe the tool surface in the $K_{2F}(x_{2F}, y_{2F}, z_{2F})$ coordinate system (see Figure 4).

The tool surface determined this way could be written as:

$$\vec{r}_{2F} = \underline{\mathbf{M}}_{2F,20}\cdot\vec{r}_{gsz}\,. \tag{6}$$

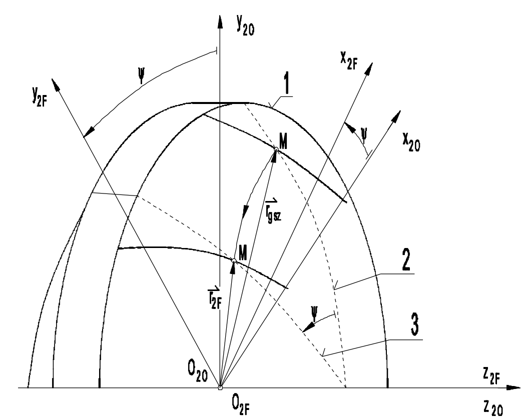


Figure 4: The tiler surface by generation curve of tool ($\vec{r}_{gsz}$) in the K_{2F} coordinate system

Where:

- 1 surface described by generating curve,
- 2 basic situation,
- 3 situation after angular displacement ψ,

$$\underline{\mathbf{M}}_{2F,20} = \begin{bmatrix} \cos\psi & -\sin\psi & 0 & 0 \\ \sin\psi & \cos\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

Therefore, the equation of the circular symmetrical tool surface is:

$$\left.\begin{aligned} x_{2F} &= x_{20}(y_{20})\cos\psi - y_{20}\sin\psi \\ y_{2F} &= x_{20}(y_{20})\sin\psi + y_{20}\cos\psi \\ z_{2F} &= z_{20}(y_{20}) \end{aligned}\right\} \qquad (8)$$

We can conclude the previously discussed facts that to determine any surface, be it either a worm surface or a tool surface, knowledge of its generating curve is necessary. In order to determine the generating curve it is enough to know some of the meshings for the other surfaces, either that of the worm or the tool surface. When the generating curve is known in the coordinate system of the surface sought (eg. in the case of a turned worm surface) than the surface can directly be determined using either equation (4) or (8) any system can be determined by transforming it into the proper coordinate system.

3. POSSIBLE USES OF THE MODEL DESCRIBED

This model is suitable for design of manufactured helicoidal surfaces with single or multiple edge tools of determined or undetermined edge geometry as well as for the design of the manufacturing tools needed. After proper choice of parameters a, b, c, γ, α (shown in Figure 2) it is an appropriate method to determine worm surface of any special profile, even non-standardized shape.
The worm surface is the relative kinematic wrapping surface of the manufacturing tool surface.
During their relative displacement the two surfaces remain in contact along a spatial curve. The general law for contact of elements is valid for any arbitrary point of this contact curve. It can be written as:

$$\vec{n}^{(1)} \cdot \vec{v}^{(12)} = \vec{n}^{(2)} \cdot \vec{v}^{(12)} = 0, \qquad (9)$$

Knowledge of the contact curve makes it possible to determine the tool surface (direct case) as well as that of the worm surface (indirect case).

3.1. *Designing the tool needed to manufacture a given worm surface (direct case)*

Given data is $\vec{r}_{1F} = \vec{r}_{1F}(\eta, \vartheta)$, the two parametric vector-scalar function in the coordinate system $K_{1F}(x_{1F}, y_{1F}, z_{1F})$ for the surface to be generated.
Let normal vector $\vec{n}_{1F}$ be determined.

$$\vec{n}_{1F} = \frac{\partial \vec{r}_{1F}}{\partial \eta} \times \frac{\partial \vec{r}_{1F}}{\partial \vartheta}. \qquad (10)$$

The relative velocity of the two surfaces can be determined in coordinate system K_{2F} using the transformation between coordinate systems K_{1F} for worm and K_{2F} for tool:

$$\vec{v}_{2F}^{(12)} = \frac{d}{dt} \cdot \vec{r}_{2F} = \frac{d}{dt}(\mathbf{\underline{M}}_{2F,1F})\, \vec{r}_{1F}. \qquad (11)$$

The vector $\vec{v}_{2F}^{(12)}$ should be transformed into coordinate system $K_{1F}(x_{1F}, y_{1F}, z_{1F})$ to determine the necessary tool surface, so:

$$\vec{v}_{1F}^{(12)} = \mathbf{\underline{M}}_{1F,2F} \cdot \vec{v}_{2F}^{(12)} = \mathbf{\underline{M}}_{1F,2F} \frac{d}{dt}(\mathbf{\underline{M}}_{2F,1F}) \cdot \vec{r}_{1F} = \mathbf{\underline{P}}_{1h} \cdot \vec{r}_{1F}, \qquad (12)$$

where

$$\mathbf{\underline{P}}_{1h} = \mathbf{\underline{M}}_{1F,2F} \cdot \frac{d}{dt}(\mathbf{\underline{M}}_{2F,1F}) \qquad (13)$$

the matrix for kinematic generation.

Solving the equation for one of its internal parameters (eg η):

$$\vec{n}_{1F}(\eta, \vartheta) \cdot \vec{v}_{1F}^{(12)}(\eta, \vartheta) = 0 \qquad (14)$$

Applying solution:

$$\vec{r}_{1F} = \vec{r}_{1F}(\eta, \vartheta) \qquad (15)$$

the equation of contact curve between surfaces is obtained in the form:

$$\vec{r}_{1F} = \vec{r}_{1F}[\eta(\vartheta), \vartheta] = \vec{r}_{1F}(\vartheta) \qquad (16)$$

which is suitable for transformation of:

$$\vec{r}_{2F}(\vartheta) = \mathbf{\underline{M}}_{2F,1F} \cdot \vec{r}_{1F}(\vartheta) \qquad (17)$$

into the tool generating system which is the generating curve of the tool. $\underline{M}_{2F,1F}$ and $\underline{M}_{1F,2F}$ are the transformation matrices between coordinate systems K_{1F} and K_{2F}.

3.2. *Determination of worm surface that can be manufactured using a given tool surface (indirect case)*

The procedure is similar to the steps carried out in the direct case, but the direction of transformation is the opposite.
Known data:

$$\vec{r}_{2F} = \vec{r}_{2F}(y_{20}, \Psi), \qquad (18)$$

$$\vec{n}_{2F} = \frac{\partial \vec{r}_{2F}}{\partial y_{20}} \times \frac{\partial \vec{r}_{2F}}{\partial \psi}, \qquad (10) \qquad (19)$$

$$\vec{v}_{2F} = P_{2h} \cdot \vec{r}_{2F}, \qquad (20)$$

where

$$P_{2h} = \underline{\mathbf{M}}_{2F,1F} \cdot \frac{d}{dt} (\underline{\mathbf{M}}_{1F,2F}) \qquad (21)$$

the matrix for kinematic generation for inverse operation.
Solving the following system of equations:

$$\vec{n}_{2F} \cdot \vec{v}_{2F} = 0 \qquad (22)$$

and

$$\vec{r}_{1F} = \underline{\mathbf{M}}_{1F,2F} \cdot \vec{r}_{2F} \qquad (23)$$

Solving these equations, enables the optiman tool profile geometry to be determined.
The position vectors $\vec{r}_{2F}$ or $\vec{r}_{1F}$ describe the searched surfaces obtained in the direct or the indirect case; these surfaces can be generated using modern CNC machine tools or traditional machine tools supplied with additional equipment.

4. MATHEMATICAL GENERATION OF HELICOIDS THREAD

The tooth surface of a conical worm, as an element of a spiroid drive, can be similarly generated as with cylindrical worms, but linked to axial displacement of the tool (p_a), depending on the measure of conical shape of the worm, a tangential displacement (p_t) of the tool is necessary as well. All this can be seen in Figures 5, 6 and 7 which show the types of helicoids and their equations too. Similarly to cylindrical worms with ruled surfaces, on the surfaces of spiroid worms several types of helicoids – involute, Archimedian or convolute – can be differentiated. But non-ruled conical worm surfaces can be investigated too.
According to present practice the teething of face gear is generated, using a worm hob with a surface equivalent to a worm wrapping surface of a conical worm. This method is called direct generation in technical literature. The involute worm gear drive with ground ruled surface has a great advantage, namely the identity of surfaces can be realized by simple machining technology. When the thread surface of the worm or worm gear milling cutter is machined using a conical or ring shape grinding wheel the precision of the profile surfaces cannot be easily guaranteed.

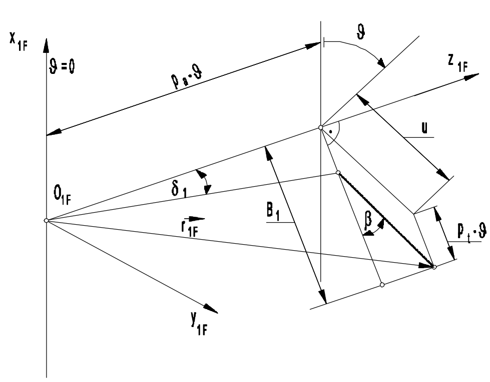


Figure 5. Generator of conical Archimedian helicoid in oblique position

The equation of conical Archimedian helicoid surface is:

$$\vec{r}_{1F} = \begin{bmatrix} -B_1 \cdot \sin \vartheta \\ +B_1 \cdot \cos \vartheta \\ u \cdot \sin \beta + p_a \cdot \vartheta \\ 1 \end{bmatrix} \qquad (24)$$

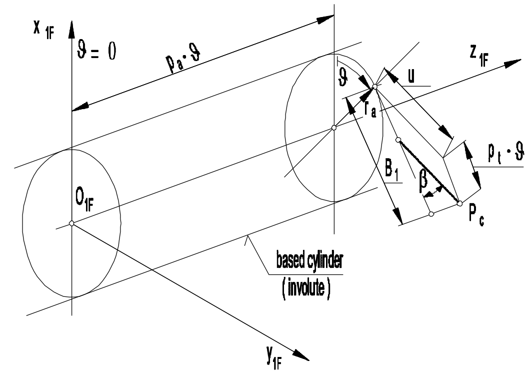


Figure 6. Generator of conical involute helicoidal surface in oblique position

The equation of conical involute helicoidal surface is:

$$\vec{r}_{1F} = \begin{bmatrix} -B_1 \cdot \sin \vartheta + r_a \cdot \cos \vartheta \\ B_1 \cdot \cos \vartheta + r_a \cdot \sin \vartheta \\ u \cdot \sin \beta + p_a \cdot \vartheta \\ 1 \end{bmatrix} \qquad (25)$$

where $r_a = p_a \cdot \mathrm{ctg}\beta - p_t$

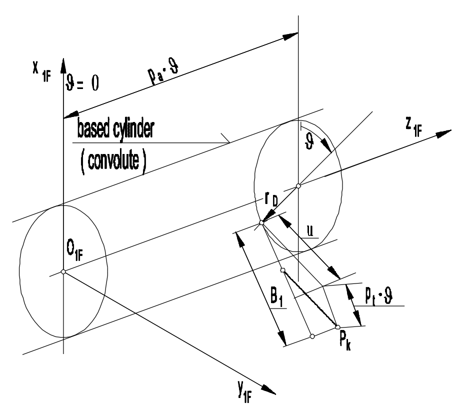


Figure 7. Generator of conical convolute helicoidal surface in oblique position

Equation of conical convolute helicoidal surface is:

$$\vec{r}_{1F} = \begin{bmatrix} -B\cdot\sin\vartheta - r_t\cdot\cos\vartheta \\ B_1\cdot\cos\vartheta - r_t\cdot\sin\vartheta \\ u\cdot\sin\beta + p_a\cdot\vartheta \\ 1 \end{bmatrix} \tag{26}$$

Regarding Figures 5, 6, and 7, it is possible to write:

$$B_1 = u\cdot\cos\beta + p_t\cdot\vartheta \tag{27}$$

$$p_t = p_a\cdot\mathrm{tg}\delta_1 \tag{28}$$

4.1. General ruled surface on conical worm

Using a suitable equation to define any oblique point fitted on any type of conical worm surface, such a formula for the position vector is obtained as can be suitable to describe the generally valid form for all the three types of conical worm surfaces (Figure 8):

$$\vec{r}_{1F} = \begin{bmatrix} -B_1\cdot\sin\vartheta + r\cdot\cos\vartheta \\ B_1\cdot\cos\vartheta + r\cdot\sin\vartheta \\ u\cdot\sin\beta + p_a\cdot\vartheta \\ 1 \end{bmatrix} \tag{29}$$

The above generally valid formula defines:

$$\left.\begin{array}{l} \text{in case of } r = 0 \text{ Archimedian} \\ r = r_a = p_a \times \mathrm{ctg}\beta - p_t > 0 \text{ involute} \\ -r_a < r = r_0 < \text{convolute helicoid surface} \end{array}\right\} \tag{30}$$

For a helicoidal surface, by substituting δ_1=0 in all three cases, the corresponding cylindrical worm is obtained.

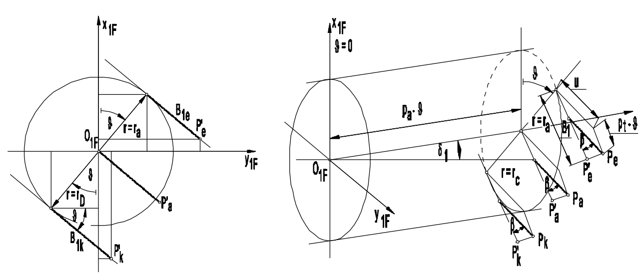


Figure 8. Summary of generation of ruled conical worm surfaces

5. REALIZATION OF THE GENERAL MATHEMATICAL MODEL

By bringing together the model created to investigate cylindrical helicoidal surfaces and their tools prepared for investigation of conical helicoidal surfaces and using appropriate parameters (γ=0) for connection analysis of drive pairs, a general kinematic model can be obtained (see Figure 9), suitable for treatment by a single mathematical model.

The transformation matrixes between the coordinate systems are:

$$\left.\begin{array}{l} \mathbf{\underline{M}}_{1,1F} = \begin{bmatrix} \cos\varphi_1 & -\sin\varphi_1 & 0 & 0 \\ \sin\varphi_1 & \cos\varphi_1 & 0 & 0 \\ 0 & 0 & 1 & -z_{ax} \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ \\ \mathbf{\underline{M}}_{1F,1} = \begin{bmatrix} \cos\varphi_1 & \sin\varphi_1 & 0 & 0 \\ -\sin\varphi_1 & \cos\varphi_1 & 0 & 0 \\ 0 & 0 & 1 & +z_{ax} \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}\right\} \tag{31}$$

$$\left.\begin{array}{l} \mathbf{\underline{M}}_{0,1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\delta_1 & \sin\delta_1 & 0 \\ 0 & -\sin\delta_1 & \cos\delta_1 & \varphi_1\cdot\sqrt{p_a^2 + p_t^2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ \\ \mathbf{\underline{M}}_{1,0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\delta_1 & -\sin\delta_1 & \varphi_1\cdot\sin\delta_1\cdot\sqrt{p_a^2 + p_t^2} \\ 0 & \sin\delta_1 & \cos\delta_1 & -\varphi_1\cdot\cos\delta_1\cdot\sqrt{p_a^2 + p_t^2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}\right\} \tag{32}$$

$$\left.\begin{array}{l} \mathbf{\underline{M}}_{2F,2} = \begin{bmatrix} \cos\varphi_2 & -\sin\varphi_2 & 0 & 0 \\ \sin\varphi_2 & \cos\varphi_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ \\ \mathbf{\underline{M}}_{2,2F} = \begin{bmatrix} \cos\varphi_2 & \sin\varphi_2 & 0 & 0 \\ -\sin\varphi_2 & \cos\varphi_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}\right\} \tag{33}$$

$$\left.\begin{array}{l} \underline{M}_{K,0} = \begin{bmatrix} \cos\gamma & 0 & \sin\gamma & -c\cdot\cos\gamma \\ 0 & 1 & 0 & a \\ -\sin\gamma & 0 & \cos\gamma & c\cdot\sin\gamma \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ \\ \mathbf{\underline{M}}_{0,K} = \begin{bmatrix} \cos\gamma & 0 & -\sin\gamma & c \\ 0 & 1 & 0 & -a \\ \sin\gamma & 0 & \cos\gamma & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}\right\} \tag{34}$$

$$\left.\begin{aligned}\underline{\mathbf{M}}_{2,K} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\gamma & \sin\gamma & 0 \\ 0 & -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ \underline{\mathbf{M}}_{K,2} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma & 0 \\ 0 & \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\end{aligned}\right\} \quad (35)$$

The direct transformation matrices between rotating coordinate systems are:

$$\underline{\mathbf{M}}_{2F,1F} = \underline{\mathbf{M}}_{2F,2} \cdot \underline{\mathbf{M}}_{2,K} \cdot \underline{\mathbf{M}}_{K,0} \cdot \underline{\mathbf{M}}_{0,1} \cdot \underline{\mathbf{M}}_{1,1F}, \quad (36)$$

$$\underline{\mathbf{M}}_{1F,2F} = \underline{\mathbf{M}}_{1F,1} \cdot \underline{\mathbf{M}}_{1,0} \cdot \underline{\mathbf{M}}_{0,K} \cdot \underline{\mathbf{M}}_{K,2} \cdot \underline{\mathbf{M}}_{2,2F}. \quad (37)$$

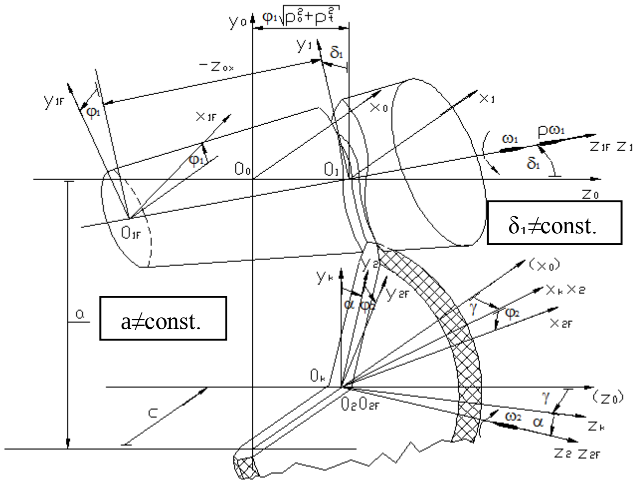


Figure 9. Correlation between coordinate systems for general investigation of machining theory of cylindrical and conical helicoidal surfaces

On Figure 9 the a and δ_1 could be changing depending of the manufacturing technology.
The solution of the direct task (surface of workpiece is known) when knowing $\vec{r}_{1F}$, the surface No2 and point of contact line is sought using equations (10), (11), (12), (13), (14), (15), (16), (17) and (18). Further, the matrices depending only on a kinematic relation can be determined as:

$$\frac{d\underline{\mathbf{M}}_{2F,1F}}{dt} \quad (38)$$

$$\underline{\mathbf{P}}_{1a} = \underline{\mathbf{M}}_{1F,2F} \cdot \frac{d\underline{\mathbf{M}}_{2F,1F}}{dt} \quad (39)$$

matrixes. P_{1a} is the matrix for the kinematic generation of the general model.
In solving the inverse problem, only the direction of transformation changes into the opposite one. Known:

$$\vec{r}_{2F} = \vec{r}_{2F}(y_{20}, \Psi). \quad (40)$$

The surface No1 according to the theory of enveloping surfaces can be derived by differentiating with respect to the movement parameter the series of surfaces generated by $\vec{r}_{2F}$ during its movement, while contact curve can be obtained by the simultaneous solution of equations:

$$\vec{n}_{2F} \cdot \vec{v}_{2F}^{21} = 0 \quad (41)$$

$$\vec{r}_{1F} = \underline{\mathbf{M}}_{1F,2F} \cdot \vec{r}_{2F} \quad (42)$$

in the K_{1F} coordinate system, where:

$$\vec{n}_{2F} = \frac{\partial \vec{r}_{2F}}{\partial y_{20}} \cdot \frac{\partial \vec{r}_{2F}}{\partial \Psi}; \quad (43)$$

(37) (37)

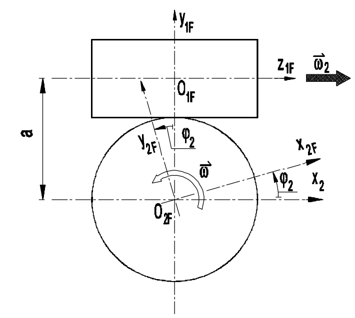
$$\vec{v}_{2F}^{(21)} = \underline{\mathbf{P}}_{2a} \cdot \vec{r}_{2F}. \quad (44)$$

To obtain the solution of this problem the P_{2a} matrix should be determined as:

$$\underline{\mathbf{P}}_{2a} = \underline{\mathbf{M}}_{2F,1F} \cdot \frac{d\underline{\mathbf{M}}_{1F,2F}}{dt}. \quad (45)$$

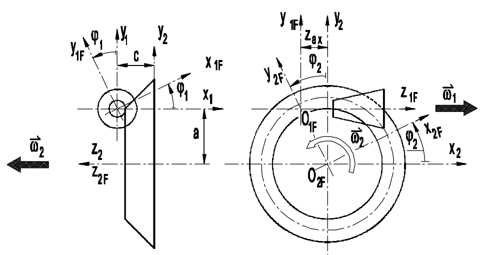
The model may be used to investigate conditions for meshing for both conical and cylindrical helicoidal surfaces with the tool having its body as the block of rotation.
Using this model it is possible to determine the contact curve starting from a known $\vec{r}_{1F}$ (No1 workpiece) (the so-called direct problem solution) as well as starting from known $\vec{r}_{2F}$ (No2 tool). We shall see further on in the text that it is also possible to start from a given contact curve as a generating curve to determine the tool surface No2 according to equation (17) and the surface of workpiece No1 as equation (22) describes it [1, 2, 3, 4, 5, 6, 7].
The surface of workpiece No1 is a cylindrical or conical basic surface having a thread curve fitted with an arbitrary generator curve (the axial section of thread).
For surface of tool No2 it is useful to define a surface of rotation, but it can be any other surface too, for example a single cutting edge tool with a cutting edge determined by φ_2=const. The frequently used types of workpiece and tool are shown in Table 1 in a tabular form giving the value of kinematic parameters which in some cases can be equal to 0.
The Figure 10 presents a typical example of the machining positions shown in Figure 10. The determination of the parameters in these two cases (Figure 10.a, b) and the presentation in the figure are found by applying the general model.
For many of the technological and kinematic positions illustrated given in Figures 10 and 11 it can be shown that the transformation ($\underline{M}_{1F,2F}$, $\underline{M}_{2F,1F}$) and the kinematic

generation matrices ($\underline{P}_{1a}$, $\underline{P}_{2a}$) created for the general model contain all possible situations.
So, by substitution of suitable parameters, matrices can be obtained both for cylindrical and for conical worm gear drives. Naturally, matrices valid for other positions can also be generated as well.



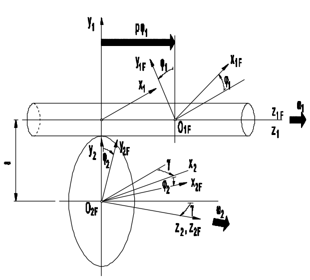
$\delta = 0$
$a > 0$
$z_{ax} = 0$
$c = 0$
$\alpha = 0°$
$\gamma = -90°$

a) Model of cylindrical worm gearing



$\delta > 0$
$a > 0$
$z_{ax} > 0$
$c > 0$
$\alpha = -90°$
$\gamma = 0°$

b) Model of spiroid worm gearing

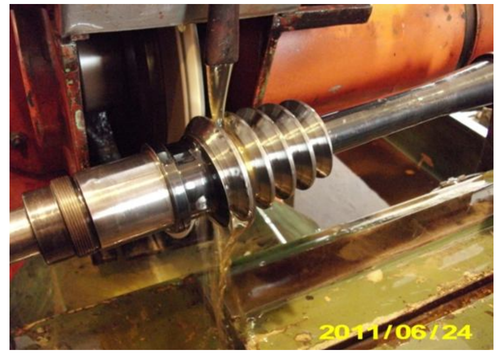


$\delta = 0$
$a > 0$
$z_{ax} = 0$
$c = 0$
$\alpha = 0°$
$\gamma > 0°$

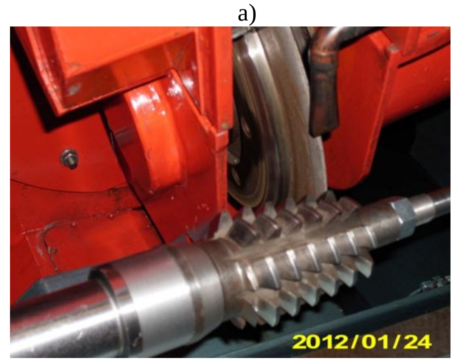
c) Grinding model of cylindrical worm

Figure 10. Main areas of application of general model [3, 4]

Using this modell worm gear drives and tools can be produced (Figure 12).



a)



b)

Figure 11. Worm (a) and tool (b) production
Difi CAD Mérnökiroda Ltd., Director: Dr. Illés Dudás, D.Sc.

CONCLUSION

I have worked out a mathematical model in which all conical and cylindrical thread surfaces can be examined based on production geometry. Changing the data of the given drive pair it is possible to examine every drive pair based on mathematical analysis and production geometry.

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APPENDIX

a, b, c		The coordinates of the origin (O_2) the tool coordinate system in the K_0 coordinate system
φ_1		Angular displacement of the helicoid (parameter for angular displacement, for meshing and for movement);
φ_2		Angular displacement of the tool (milling cutter or grinding wheel);
d_{a1}	(mm)	Addendum cylinder diameter of the worm
d_{g1}	(mm)	Pitch cylinder diameter of the worm
d_{fl}	(mm)	Root cylinder diameter of the worm
h_{f1}	(mm)	Dedendum height of the worm tooth
h_{a1}	(mm)	Addendum height of the worm tooth
K_0 (x_0, y_0, z_0)		Stationary coordinate system affixed to machine tool
K_{1F} (x_{1F}, y_{1F},z_{1F})		Rotating coordinate system affixed helicoid surface
K_{2F} (x_{2F}, y_{2F},z_{2F})		Rotating coordinate system fixed to worm gear
Ks (ξ,η,ζ)		Tool coordinate system of generating curve of helicoid surface
m	(mm)	Axial module
$M_{lF,2F}$		Coordinate transformation matrix (transforms K_{2F} to K_{1F})
$M_{2F,1F}$		Coordinate transformation matrix (transforms K_{1F} to K_{2F})
n_{1F}		Unit normal vector of helicoid surface in coordinate system K_{1F}
n_{2F}		Unit normal vector of tool surface in coordinate system K_{2F}
O_0, O_1, O_2, O_{1F}, O_{2F}		Origins of coordinate systems related to their subscripts
p		Screw parameter of the helix on worm
p_t		Tangential screw parameter
p_a		Axial screw parameter
P_{1h}, P_{1k}, P_{1s}, P_{1a}		Kinematic projection matrix, for direct method (cylindrical, conical, general)
p_x	(mm)	Axial pitch of the worm
p_z	(mm)	Lead of thread
S_1	(mm)	Tooth thickness of the worm
S_{1F}	(mm)	Tooth thickness of dedendum of the tooth of the worm
$v_{1F(1,2)}$	(m/ min-1)	Velocity vector of helicoid and tool surfaces in the K_{1F} coordinate system
$v_{2F(1,2)}$	(m/ min-1)	Velocity vector of helicoid and tool surfaces in the K_{2F} coordinate system
ρ_{ax}	(mm)	Radius of tooth profile of worm having circular profile in axial section
ζ,η,ξ		Axes of the coordinate system (K_{sz}) of the tool
i_{21}		$i_{21} = \varphi_2/\varphi_1$ gearing ratio;
$\vec{r}_{sz}$		The equation of the tool edge or generating curve in the tool coordinate system K_{sz}
$\vec{r}_{1F}$		The position vector of a point fitted on worm surface
$\vec{r}_{2F}$		The position vector of an oblique point fitted on tool surface
γ_o		Lead angle on worm reference cylinder
α		The inclination of the tool to the profile of the helicoid measured in characteristic section (eg grinding of involute worm using plane flank surface tool);
z_1		Number of teeth on worm

Table 9. The most frequently used types of workpiece and tool surfaces characterized by general model parameters

Movement geometrical feature			a	c	α	γ	δ	P_a	P_r
Tool type is 2	Type of workpiece 1.								
Disc type milling	Cylindrical worm	ZA	>0	0	0	≠0	0	≠0	0
Disc type milling	Cylindrical worm	ZI*	>0	0	0	≠0	0	≠0	0
Disc type milling	Cylindrical worm	ZI**	>0	≠0	>0	≠0	0	≠0	0
Disc type milling	Cylindrical worm	ZN	>0	0	0	≠0	0	≠0	0
Disc type milling	Cylindrical worm	ZT	>0	0	0	≠0	0	≠0	0
Disc type milling	Cylindrical worm	ZK	0	0	0	≠0	0	≠0	0
Disc type milling	Conical worm	KA	>0	0	0	≠0	>0	≠0	>0
Disc type milling	Conical worm	KI*	>0	0	0	≠0	>0	≠0	>0
Disc type milling	Conical worm	KI**	>0	≠0	>0	≠0	>0	≠0	>0
Disc type milling	Conical worm	KN	>0	0	0	≠0	>0	≠0	>0
Disc type milling	Conical worm	KT	>0	0	0	≠0	>0	≠0	>0
Disc type milling	Conical worm	KK	>0	0	0	≠0	>0	≠0	>0
Disc type milling	Axial flank surface		>0	0	0	≠0	0	≠0	0
Disc type milling	Radial and diagonal flank surface		>0	0	0	≠0	>0	≠0	>0
Pin type grinding	Cylindrical worm	ZA	>0	0	-90°	0	0	≠0	0
Pin type grinding	Cylindrical worm	ZI*	>0	0	-90°	0	0	≠0	0
Pin type grinding	Cylindrical worm	ZI**	-	-	-	-	-	-	-
Pin type grinding	Cylindrical worm	ZN	>0	0	-90°	0	0	≠0	0
Pin type grinding	Cylindrical worm	ZT	>0	0	-90°	0	0	≠0	0
Pin type grinding	Cylindrical worm	ZK	>0	0	-90°	0	0	≠0	0
Pin type grinding	Conical worm	KA	>0	0	-90°	0	>0	≠0	>0
Pin type grinding	Conical worm	KI*	>0	0	-90°	0	>0	≠0	>0
Pin type grinding	Conical worm	KI**	-	-	-	-	-	-	-
Pin type grinding	Conical worm	KN	>0	0	-90°	0	>0	≠0	>0
Pin type grinding	Conical worm	KT	>0	0	-90°	0	>0	≠0	>0
Pin type grinding	Conical worm	KK	>0	0	-90°	0	0	≠0	0
Pin type grinding	Axial flank surface		0	0	-90°	0	0	≠0	0
Pin type grinding	Radial and diagonal flank surface		0	0	-90°	0	>0	≠0	>0
Straight cup	Cylindrical worm	ZA	>0	≠0	>0	≠0	>0	≠0	0
Straight cup	Cylindrical worm	ZI*	>0	≠0	>0	≠0	0	≠0	0
Straight cup	Cylindrical worm	ZI**	-	-	-	-	-	-	-
Straight cup	Cylindrical worm	ZN	>0	≠0	>0	≠0	0	≠0	0
Straight cup	Cylindrical worm	ZT	>0	≠0	>0	≠0	0	≠0	0
Straight cup	Cylindrical worm	ZK	>0	≠0	>0	≠0	0	≠0	0
Straight cup	Conical worm	KA	>0	≠0	>0	≠0	>0	≠0	>0
Straight cup	Conical worm	KI*	>0	≠0	>0	≠0	>0	≠0	>0
Straight cup	Conical worm	KI**	-	-	-	-	-	-	-
Straight cup	Conical worm	KN	>0	≠0	>0	≠0	>0	≠0	>0
Straight cup	Conical worm	KT	>0	≠0	>0	≠0	>0	≠0	>0
Straight cup	Conical worm	KK	>0	≠0	>0	≠0	>0	≠0	>0
Straight cup	Axial flank surface		>0	≠0	>0	≠0	0	≠0	0
Straight cup	Radial and diagonal flank surface		>0	≠0	>0	≠0	>0	≠0	>0