GENERAL MATHEMATICAL MODEL FOR INVESTIGATION OF CYLINDRICAL AND CONICAL WORMS, WORM GEARS AND FACE GEARS

Prof. Dr. Illés Dudás
D.Sc., professor emeritus
Institute of Manufacturing Science,
University of Miskolc, H-3515
Miskolc, Egyetemváros, Hungary,

ABSTRACT
The objective of this publication is based on the results of kinematical geometry and toothing theory generalization of geometric correct production of worm surfaces (e.g. turning, milling, grinding), production geometry analysis of tools, the mathematical defining of the geometric and connection relation. Our purpose is to be able to define every thread surface on one common system so that they could be produced in a modern manufacturing system.

Keywords
conical worms, face gear, hob

1. INTRODUCTION
One the most modern types of cylindrical helicoidal surfaces is the worm generated using a circular profile tool.

2. INVESTIGATION OF GEOMETRIC PROBLEMS WHEN MANUFACTURING CYLINDRICAL HELICOID SURFACES USING GENERAL MATHEMATICAL-KINEMATIC MODELS
First, $P_1$, the general transformation matrix, should be determined, thus solving the connection between the coordinate systems belonging to the generating tool and the generated surface, including the kinematics of generation. Coordinate systems used in this investigation and their relationships are shown in Figure 2.

The investigation of geometric problems related to the manufacture of cylindrical helicoid surfaces is best carried out in a general system. This general system makes it possible to discuss cylindrical worm surfaces, their different manufacturing methods and generating tools.

During our investigations the kinematics of generation was handled so that the helicoid surface followed a thread path and the
tool surface performed a rotary motion on the left side of the thread profiles; the lead of thread and generator curve together with signs should be taken into consideration.

When discussing geometric problems of manufacturing in general it is necessary to determine generally valid rules for generation of the cylindrical thread surface [5].

The position vector \( \vec{g}_r \) of the generating curve in the coordinate system \( \mathbf{K}_S(\xi, \eta, \zeta) \) is given as a function. This generating curve can be the edge of tool (e.g., in lathe turning) or the contact curve (e.g., in grinding) [4]. To formulate the equation of the generating curve, from the practical point of view let the parameter \( \eta \) be chosen as an independent variable. In this way the parametric vector function of the generating curve is found to be:

\[
\vec{g}_r = \xi(\eta) \vec{i} + \eta \vec{j} + \zeta(\eta) \vec{k}.
\] (1)

The generating curve parametric equation \( \vec{g}_r \) is carried by the generating curve parametric equation \( \mathbf{K}_S(\xi, \eta, \zeta) \) the coordinate system is forced on the thread path along axis \( \zeta \) with parameter \( p \), so the generating curve will describe a thread surface in coordinate system \( \mathbf{K}_1F(x_1F, y_1F, z_1F) \) which, performed before this movement, coincides with the \( \mathbf{K}_S \) coordinate system (see Figure 3).

![Figure 3: The generating curve of the thread surface in \( \mathbf{K}_{1F} \) coordinate system](image)

The thread surface described by the generating curve \( \vec{g}_r \) can be determined in \( \mathbf{K}_{1F}(x_{1F}, y_{1F}, z_{1F}) \) coordinate system as:

\[
\vec{r}_{1F} = \mathbf{M}_{1F, S} \vec{g}_r.
\] (2)

\[
\mathbf{M}_{1F, S} = \begin{bmatrix}
\cos \varphi & -\sin \varphi & 0 & 0 \\
\sin \varphi & \cos \varphi & 0 & 0 \\
0 & 0 & 1 & p \cdot \varphi \\
0 & 0 & 0 & 0
\end{bmatrix}.
\] (3)

Therefore, the general equation of the cylindrical worm surface is:

\[
\begin{align*}
x_{1F} &= \xi(\eta) \cdot \cos \vartheta - \eta \cdot \sin \vartheta \\
y_{1F} &= \xi(\eta) \cdot \sin \vartheta + \eta \cdot \cos \vartheta \\
z_{1F} &= \xi(\eta) + p \cdot \vartheta.
\end{align*}
\] (4)

It can be seen from the structure of transformation matrix \( \mathbf{M}_{1F, S} \) and the general basically equation of the worm surface (3) that the generating curve \( \vec{g}_r \) and worm parameter \( p \) determine the worm surface.

The generator curve \( \vec{g}_r \) has a decisive role in the case of tool surface generation too. During generation of the tool surface, the generator curve can be the meridian curve or the contact curve. In this case the \( \vec{g}_{st} \) curve is interpreted in the \( \mathbf{K}_{20}(x_{20}, y_{20}, z_{20}) \) coordinate system using \( y_{20} \) as a parameter, so its form is:

\[
\vec{g}_{st} = x_{20}(y_{20}) \vec{i} + y_{20} \vec{j} + z_{20}(y_{20}) \vec{k}.
\] (5)

Rotating \( \vec{g}_{st} \) generating curve with the \( \mathbf{K}_{20}(x_{20}, y_{20}, z_{20}) \) coordinate system round the \( z_{20} \) axis the \( \vec{r}_{st} \) curve will describe the tool surface in the \( \mathbf{K}_{2F}(x_{2F}, y_{2F}, z_{2F}) \) coordinate system (see Figure 4).

The tool surface determined this way could be written as:

\[
\vec{r}_{2F} = \mathbf{M}_{2F, 20} \vec{r}_{st}.
\] (6)

![Figure 4: The tiler surface by generation curve of tool (\( \vec{r}_{st} \)) in the \( \mathbf{K}_{2F} \) coordinate system](image)

Where:
- 1 surface described by generating curve,
- 2 basic situation,
- 3 situation after angular displacement \( \psi_r \).

\[
\mathbf{M}_{2F, 20} = \begin{bmatrix}
\cos \psi_r & -\sin \psi_r & 0 & 0 \\
\sin \psi_r & \cos \psi_r & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}.
\] (7)
Therefore, the equation of the circular symmetrical tool surface is:

\[
\begin{align*}
    x_{1F} &= x_{20}(y_{20})\cos\psi - y_{20}\sin\psi \\
    y_{1F} &= x_{20}(y_{20})\sin\psi + y_{20}\cos\psi \\
    z_{1F} &= z_{20}(y_{20})
\end{align*}
\]  

\[ (8) \]

We can conclude the previously discussed facts that to determine any surface, be it a worm surface or a tool surface, knowledge of its generating curve is necessary. In order to determine the generating curve it is enough to know some of the meshes for the other surfaces, either that of the worm or the tool surface. When the generating curve is known in the coordinate system of the surface sought (eg. in the case of a turned worm surface) than the surface can directly be determined using either equation (4) or (8) any system can be determined by transforming it into the proper coordinate system.

3. POSSIBLE USES OF THE MODEL DESCRIBED

This model is suitable for design of manufactured helicoidal surfaces with single or multiple edge tools of determined or undetermined edge geometry as well as for the design of the manufacturing tools needed. After proper choice of parameters a, b, c, \( \gamma \), \( \alpha \) (shown in Figure 2) it is an appropriate method to determine worm surface of any special profile, even non-standardized shape.

The worm surface is the relative kinematic wrapping surface of the tool surface. During their relative displacement the two surfaces remain in contact along a spatial curve. The general law for contact of elements is valid for any arbitrary point of this contact curve. It can be written as:

\[
\vec{n}^{(1)} \cdot \vec{v}^{(12)} = \vec{n}^{(2)} \cdot \vec{v}^{(12)} = 0, 
\]

\[ (9) \]

Knowledge of the contact curve makes it possible to determine the tool surface (direct case) as well as that of the worm surface (indirect case).

3.1. Designing the tool needed to manufacture a given worm surface (direct case)

Given data is \( \vec{r}_{1F} = \vec{r}_{1F}(\eta, \vartheta) \), the two parametric vector-scalar function in the coordinate system \( K_{1F}(x_{1F}, y_{1F}, z_{1F}) \) for the surface to be generated.

Let normal vector \( \vec{n}_{1F} \) be determined.

\[
\vec{n}_{1F} = \frac{\partial \vec{r}_{1F}}{\partial \eta} \times \frac{\partial \vec{r}_{1F}}{\partial \vartheta},
\]

\[ (10) \]

The relative velocity of the two surfaces can be determined in coordinate system \( K_{2F} \) using the transformation between coordinate systems \( K_{1F} \) for worm and \( K_{2F} \) for tool:

\[
\vec{v}_{2F}^{(12)} = \frac{d}{dt}\vec{r}_{2F} = \frac{d}{dt}(\frac{M_{2F,1F}}{dt}) \vec{r}_{1F},
\]

\[ (11) \]

The vector \( \vec{v}_{2F}^{(12)} \) should be transformed into coordinate system \( K_{1F}(x_{1F}, y_{1F}, z_{1F}) \) to determine the necessary tool surface, so:

\[
\vec{v}_{1F}^{(12)} = \frac{d}{dt}\vec{r}_{1F} = \frac{d}{dt}(M_{2F,1F})\vec{r}_{1F} = P_{1F}\vec{r}_{1F},
\]

\[ (12) \]

where

\[
P_{1F} = \frac{d}{dt}(M_{2F,1F})
\]

the matrix for kinematic generation.

Solving the equation for one of its internal parameters (eg \( \eta \)):

\[
\vec{n}_{1F}(\eta, \vartheta) \cdot \vec{v}_{1F}^{(12)}(\eta, \vartheta) = 0
\]

\[ (13) \]

Applying solution:

\[
\vec{n}_{1F}(\eta, \vartheta) = \vec{n}_{1F}(\eta, \vartheta)
\]

\[ (14) \]

the equation of contact curve between surfaces is obtained in the form:

\[
\vec{r}_{1F} = \vec{r}_{1F}(\eta, \vartheta)
\]

\[ (15) \]

which is suitable for transformation of:

\[
\vec{r}_{1F}(\vartheta) = \frac{d}{d\vartheta}(M_{2F,1F})\vec{r}_{1F}(\vartheta)
\]

\[ (16) \]

into the tool generating system which is the generating curve of the tool. \( M_{2F,1F} \) and \( M_{2F,2F} \) are the transformation matrices between coordinate systems \( K_{1F} \) and \( K_{2F} \).

3.2. Determination of worm surface that can be manufactured using a given tool surface (indirect case)

The procedure is similar to the steps carried out in the direct case, but the direction of transformation is the opposite.

Known data:

\[
\vec{r}_{1F} = \vec{r}_{1F}(\eta, \vartheta),
\]

\[ (17) \]

\[
\frac{\partial \vec{r}_{1F}}{\partial \vartheta} = \vec{r}_{1F}(\vartheta, \eta)
\]

\[ (18) \]

\[
\vec{n}_{2F} = \vec{n}_{2F}(\vartheta, \psi),
\]

\[ (19) \]

\[
\vec{v}_{2F} = P_{2F}\vec{r}_{2F},
\]

\[ (20) \]
GENERAL MATHEMATICAL MODEL FOR INVESTIGATION OF CYLINDRICAL AND CONICAL WORMS, WORM GEARS AND FACE GEARS

\[ P_{2h} = \mathbf{M}_{2p,1} \frac{d}{dt} (\mathbf{M}_{1,2p}) \]  \hspace{1cm} (21)

the matrix for kinematic generation for inverse operation.

Solving the following system of equations:

\[ \mathbf{n}_{2p} \cdot \mathbf{v}_{2p} = 0 \]  \hspace{1cm} (22)

and

\[ \mathbf{r}_{2p} = \mathbf{M}_{1,2p} \mathbf{r}_{2p} \]  \hspace{1cm} (23)

Solving these equations, enables the optiman tool profile geometry to be determined.

The position vectors \( \mathbf{r}_{2p} \) or \( \mathbf{r}_{1p} \) describe the searched surfaces obtained in the direct or the indirect case; these surfaces can be generated using modern CNC machine tools or traditional machine tools supplied with additional equipment.

4. MATHEMATICAL GENERATION OF HELICOID THREAD

The tooth surface of a conical worm, as an element of a spiroid drive, can be similarly generated as with cylindrical worms, but linked to axial displacement of the tool (\( p_a \)), depending on the measure of conical shape of the worm, a tangential displacement (\( p_t \)) of the tool is necessary as well. All this can be seen in Figures 5, 6 and 7 which show the types of helicoids and their equations too. Similarly to cylindrical worms with ruled surfaces, on the surfaces of spiroid worms several types of helicoids – involute, Archimedian or convolute – can be differentiated. But non-ruled conical worm surfaces can be investigated too.

According to present practice the teething of face gear is generated, using a worm hob with a surface equivalent to a worm wrapping surface of a conical worm. This method is called direct generation in technical literature. The involute worm gear drive with ground ruled surface has a great advantage, namely the identity of surfaces can be realized by simple machining technology. When the thread surface of the worm or worm gear milling cutter is machined using a conical or ring shape grinding wheel the precision of the profile surfaces cannot be easily guaranteed.

The equation of conical Archimedian helicoid surface is:

\[ \mathbf{r}_{2p} = \begin{bmatrix} -B_i \cdot \sin \vartheta \\ +B_i \cdot \cos \vartheta \\ u \cdot \sin \beta + p_a \cdot \vartheta \end{bmatrix} \]  \hspace{1cm} (24)

Figure 6. Generator of conical involute helicoidal surface in oblique position

The equation of conical involute helicoidal surface is:

\[ \mathbf{r}_{2p} = \begin{bmatrix} -B_i \cdot \sin \vartheta + r_a \cdot \cos \vartheta \\ +B_i \cdot \cos \vartheta + r_a \cdot \sin \vartheta \\ u \cdot \sin \beta + p_a \cdot \vartheta \end{bmatrix} \]  \hspace{1cm} (25)

where \( r_a = p_a \cdot \cot \beta - p_t \).

Figure 7. Generator of conical convolute helicoidal surface in oblique position

Equation of conical convolute helicoidal surface is:
\[
\vec{r}_{\alpha} = \begin{bmatrix}
-B \cdot \sin \beta - r \cdot \cos \beta \\
B \cdot \cos \beta - r \cdot \sin \beta \\
u \cdot \sin \beta + p_a \cdot \beta \\
l
\end{bmatrix}
\] (26)

Regarding Figures 5, 6, and 7, it is possible to write:

\[B_1 = u \cdot \cos \beta + p_i \cdot \beta\] (27)

\[p_i = p_i \cdot \alpha \cdot \delta_i\] (28)

4.1. General ruled surface on conical worm

Using a suitable equation to define any oblique point fitted on any type of conical worm surface, such a formula for the position vector is obtained as can be suitable to describe the generally valid form for all the three types of conical worm surfaces (Figure 8):

\[
\vec{r}_{\alpha} = \begin{bmatrix}
-B_1 \cdot \sin \beta + r \cdot \cos \beta \\
B_1 \cdot \cos \beta + r \cdot \sin \beta \\
u \cdot \sin \beta + p_a \cdot \beta \\
l
\end{bmatrix}
\] (29)

The above generally valid formula defines:

\[
\begin{align*}
\text{in case of } r = 0 & \text{ Archimedian} \\
r = r_e = p_e \cdot \alpha \cdot \beta \cdot \delta_i > 0 & \text{ involute} \\
- r < r < r_e < \text{ convolute helicoid surface}
\end{align*}
\] (30)

For a helicoidal surface, by substituting \(\delta_i=0\) in all three cases, the corresponding cylindrical worm is obtained.

The transformation matrixes between the coordinate systems are:

\[
M_{1,1} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \delta & \sin \delta & 0 \\
0 & -\sin \delta & \cos \delta & \sqrt{p_a^2 + p_i^2} \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (31)

\[
M_{1,1} = \begin{bmatrix}
\cos \phi & -\sin \phi & 0 & 0 \\
\sin \phi & \cos \phi & 0 & 0 \\
0 & 0 & 1 & -z_{ax} \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
M_{1,0} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \delta & \sin \delta & 0 \\
0 & -\sin \delta & \cos \delta & \sqrt{p_a^2 + p_i^2} \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
M_{1,0} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \delta & \sin \delta & \phi \cdot \sqrt{p_a^2 + p_i^2} \\
0 & -\sin \delta & \cos \delta & \sqrt{p_a^2 + p_i^2} \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

5. REALIZATION OF THE GENERAL MATHEMATICAL MODEL

By bringing together the model created to investigate cylindrical helicoidal surfaces and their tools prepared for investigation of conical helicoidal surfaces and using appropriate parameters (\(\gamma=0\)) for connection analysis of drive pairs, a general kinematic model can be obtained (see Figure 9), suitable for treatment by a single mathematical model.
The direct transformation matrices between rotating coordinate systems are:

\[
\begin{align*}
M_{1,F} &= M_{2,F} M_{1,F} M_{0,F} M_{0,1} M_{1,F}, \\
M_{1,F,2} &= M_{1,F,1} M_{1,2} M_{2,2} M_{2,F},
\end{align*}
\]

(36)

(37)

Figure 9. Correlation between coordinate systems for general investigation of machining theory of cylindrical and conical helicoidal surfaces

On Figure 9 the \(a\) and \(\delta_f\) could be changing depending on the manufacturing technology. The solution of the direct task (surface of workpiece is known) when knowing \(r_{1F}\), the surface No2 and point of contact line is sought using equations (10), (11), (12), (13), (14), (15), (16), (17) and (18). Further, the matrices depending only on a kinematic relation can be determined as:

\[
\begin{align*}
\frac{dM_{1,F,3F}}{dt} = f(F), \\
P_{2a} = M_{2,F,2F} \frac{dM_{1,F,1F}}{dt},
\end{align*}
\]

(38)

(39)

matrices. \(P_{2a}\) is the matrix for the kinematic generation of the general model.

In solving the inverse problem, only the direction of transformation changes into the opposite one. Known:

\[
\bar{r}_{1F} = \bar{r}_{1F}(y^{20},\gamma').
\]

(40)

The surface No1 according to the theory of enveloping surfaces can be derived by differentiating with respect to the movement parameter the series of surfaces generated by \(\bar{r}_{1F}\) during its movement, while contact curve can be obtained by the simultaneous solution of equations:

\[
\begin{align*}
\bar{n}_{2F} \cdot \vec{v}_{2F}^{(21)} &= 0, \\
\bar{r}_{2F} &= M_{2,F,2F} \cdot \bar{r}_{2F},
\end{align*}
\]

(41)

(42)

in the \(K_{1F}\) coordinate system, where:

\[
\begin{align*}
\bar{n}_{2F} &= \frac{\partial \bar{r}_{2F}}{\partial y_{20}} \cdot \frac{\partial \bar{r}_{2F}}{\partial \gamma'} (37) \\
\vec{v}_{2F}^{(21)} &= \frac{d}{dt} \bar{r}_{2F},
\end{align*}
\]

(43)

(44)

To obtain the solution of this problem the \(P_{2a}\) matrix should be determined as:

\[
P_{2a} = M_{2,F,3F} \cdot \frac{dM_{1,F,2F}}{dt}.
\]

(45)

The model may be used to investigate conditions for meshing for both conical and cylindrical helicoidal surfaces with the tool having its body as the block of rotation. Using this model it is possible to determine the contact curve starting from a known \(r_{1F}\) (No1 workpiece) (the so-called direct problem solution) as well as starting from known \(r_{2F}\) (No2 tool). We shall see further on in the text that it is also possible to start from a given contact curve as a generating curve to determine the tool surface No2 according to equation (17) and the surface of workpiece No1 as equation (22) describes it [1, 2, 3, 4, 5, 6, 7].

The surface of workpiece No1 is a cylindrical or conical basic surface having a thread curve fitted with an arbitrary generator curve (the axial section of thread). For surface of tool No2 it is useful to define a surface of rotation, but it can be any other surface too, for example a single cutting edge tool with a cutting edge determined by \(\phi_2=\text{const}\). The frequently used types of workpiece and tool are shown in Table 1 in a tabular form giving the value of kinematic parameters which in some cases can be equal to 0.

The Figure 10 presents a typical example of the machining positions shown in Figure 10. The determination of the parameters in these two cases (Figure 10.a, b) and the presentation in the figure are found by applying the general model. For many of the technological and kinematic positions illustrated given in Figures 10 and 11 it can be shown that the transformation \((M_{1,F,2F}, M_{2,F,1F})\) and the kinematic
generation matrices \( (P_{1a}, P_{2a}) \) created for the general model contain all possible situations. So, by substitution of suitable parameters, matrices can be obtained both for cylindrical and for conical worm gear drives. Naturally, matrices valid for other positions can also be generated as well.

![Diagram of worm gear](image1)

a) Model of cylindrical worm gearing

\[
\begin{align*}
\delta &= 0 \\
a &> 0 \\
z_{ai} &= 0 \\
c &= 0 \\
\alpha &= 0^\circ \\
\gamma &= -90^\circ
\end{align*}
\]

![Diagram of spiroid gear](image2)

b) Model of spiroid worm gearing

\[
\begin{align*}
\delta &= 0 \\
a &> 0 \\
z_{ai} &> 0 \\
c &> 0 \\
\alpha &= -90^\circ \\
\gamma &= 0^\circ
\end{align*}
\]

c) Grinding model of cylindrical worm

Figure 10. Main areas of application of general model [3, 4]

Using this model, worm gear drives and tools can be produced (Figure 12).

![Worm and tool production](image3)

Figure 11. Worm (a) and tool (b) production

DIFFI CAD MÉRNÖKIRODA LTD., DIRECTOR: DR. ILLÉS DUDÁS, D.SC.

CONCLUSION

I have worked out a mathematical model in which all conical and cylindrical thread surfaces can be examined based on production geometry. Changing the data of the given drive pair it is possible to examine every drive pair based on mathematical analysis and production geometry.

REFERENCES

GENERAL MATHEMATICAL MODEL FOR INVESTIGATION OF CYLINDRICAL AND CONICAL WORMS, WORM GEARS AND FACE GEARS

APPENDIX

| $a, b, c$ | The coordinates of the origin ($O_2$) the tool coordinate system in the $K_0$ coordinate system |
| $\varphi_1$ | Angular displacement of the helicoid (parameter for angular displacement, for meshing and for movement); |
| $\varphi_2$ | Angular displacement of the tool (milling cutter or grinding wheel); |
| $d_{a1}$ (mm) | Addendum cylinder diameter of the worm |
| $d_{g1}$ (mm) | Pitch cylinder diameter of the worm |
| $d_{fl}$ (mm) | Root cylinder diameter of the worm |
| $h_{f1}$ (mm) | Dedendum height of the worm tooth |
| $h_{a1}$ (mm) | Addendum height of the worm tooth |
| $K_0$ ($x_0, y_0, z_0$) | Stationary coordinate system affixed to machine tool |
| $K_{1F}$ ($x_{1F}, y_{1F}, z_{1F}$) | Rotating coordinate system affixed helicoid surface |
| $K_{2F}$ ($x_{2F}, y_{2F}, z_{2F}$) | Rotating coordinate system fixed to worm gear |
| $K_S$ ($\xi, \eta, \zeta$) | Tool coordinate system of generating curve of helicoid surface |
| $m$ (mm) | Axial module |
| $M_{2F,1F}$ | Coordinate transformation matrix (transforms $K_{2F}$ to $K_{1F}$) |
| $M_{3F,1F}$ | Coordinate transformation matrix (transforms $K_{3F}$ to $K_{1F}$) |
| $n_{1F}$ | Unit normal vector of helicoid surface in coordinate system $K_{1F}$ |
| $n_{2F}$ | Unit normal vector of tool surface in coordinate system $K_{2F}$ |
| $O_{0}, O_{1}, O_{2}, O_{1F}, O_{2F}$ | Origins of coordinate systems related to their subscripts |
| $p$ | Screw parameter of the helix on worm |
| $p_t$ | Tangential screw parameter |
| $p_a$ | Axial screw parameter |
| $P_{III}$, $P_{I_{11}}, P_{I_{11}}$ | Kinematic projection matrix, for direct method (cylindrical, conical, general) |
| $p_x$ (mm) | Axial pitch of the worm |
| $p_z$ (mm) | Lead of thread |
| $S_1$ (mm) | Tooth thickness of the worm |
| $S_{1F}$ (mm) | Tooth thickness of dedendum of the tooth of the worm |
| $v_{1F(1,2)}$ (m/min) | Velocity vector of helicoid and tool surfaces in the $K_{1F}$ coordinate system |
| $v_{2F(1,2)}$ (m/min) | Velocity vector of helicoid and tool surfaces in the $K_{2F}$ coordinate system |
| $\rho_{ax}$ (mm) | Radius of tooth profile of worm having circular profile in axial section |
| $\xi, \eta, \zeta$ | Axes of the coordinate system ($K_{ax}$) of the tool |
| $i_{z1}$ | $i_{z1} = \varphi_2 / \varphi_1$ gearing ratio; |
| $\bar{r}_{sz}$ | The equation of the tool edge or generating curve in the tool coordinate system $K_{ax}$ |
| $\bar{r}_{1F}$ | The position vector of a point fitted on worm surface |
| $\bar{r}_{2F}$ | The position vector of an oblique point fitted on tool surface |
| $\gamma_0$ | Lead angle on worm reference cylinder |
| $\alpha$ | The inclination of the tool to the profile of the helicoid measured in characteristic section (eg grinding of involute worm using plane flank surface tool); |
| $z_1$ | Number of teeth on worm |
Table 9. The most frequently used types of workpiece and tool surfaces characterized by general model parameters

<table>
<thead>
<tr>
<th>Movement geometrical feature</th>
<th>Type of workpiece 1.</th>
<th>Type of workpiece 2.</th>
<th>Tool type is 2</th>
<th>Type of workpiece 3.</th>
<th>Type of workpiece 4.</th>
<th>Type of workpiece 5.</th>
<th>Type of workpiece 6.</th>
<th>Type of workpiece 7.</th>
<th>Type of workpiece 8.</th>
<th>Type of workpiece 9.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disc type milling</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cylindrical worm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conical worm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pin type grinding</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cylindrical worm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conical worm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Straight cup</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cylindrical worm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conical worm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

International Journal of Innovative Research in Engineering & Management (IJIREM)
ISSN: 2350-0557, Volume-2, Issue-5, September-2015